Microwaves

Series 7, solutions

Problem 1

Find the properties of the following 5 two-port components, given by their scattering matrix. Are they reciprocal, lossless, matched, symmetrical ? What kind of components are they ?

a) $\begin{bmatrix} 0 & 250 \\ 0,2 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 0,6 & j0,8 \\ j0,8 & 0,6 \end{bmatrix}$ c) $\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 0,1 \\ 0,1 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 1,5 & j0,1 \\ j0,5 & 0 \end{bmatrix}$

1. All these components are LINEAR
2. Are they RECIPROCAL ?

a) NO	b) YES	c) NO	d) YES	e) NO

3. Are they LOSSLESS ?

a) NO (active)	b) YES	c) YES	d) NO	e) NO (active)

4. Are they MATCHED ?

a) YES	b) NO	c) YES	d) YES	e) NO

5. Are they SYMMETRICAL ?

a) NO	b) YES	c) NO	d) YES	e) NO

a) amplifier	b) obstacle	c) gyrator	d) attenuator	e) ????

HOW DO THEY WORK ?

a) When a signal is applied at port 2, the amplitude of the signal flowing out of 1 is 250 times larger. This component produces thus an amplification. In the other direction, a small part of the signal flows. The amplifier is thus not unilateral, and there might be an interaction between the input and output circuit of the amplifiers (which we usually do not like).

b) The signal incoming on one port is partly reflected and partly transmitted. Nothing is dissipated. We have here an unmatched attenuator (reactive attenuator) or an obstacle used for matching purposes for instance.

c) This device transfers integrally the received signal to the other port, but with a phase shift which depends on the direction. It is a non reciprocal device, called gyrator.

d) This component is matched, but transfer only a tenth of the signal to the output (or the hundredth part of the power). It is an attenuator of 20 dB absorption.

e) This component produced a reflection greater than 1 at port 1m there is thus an amplification. But there is also a transmission to port 2, so this is a rather strange element.

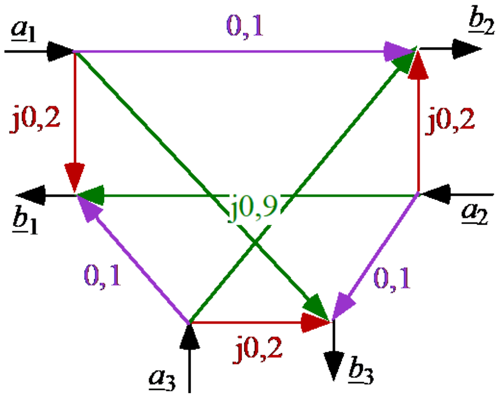
Problem 2
A three port circuit is characterized by the following scattering matrix :

$$\begin{bmatrix} j0,2 & j0,9 & 0,1 \\ 0,1 & j0,2 & j0,9 \\ j0,9 & 0,1 & j0,2 \end{bmatrix}$$

Draw its flowchart, end determine if it is a linear, reciprocal, lossless, matched element. What could be its use ?

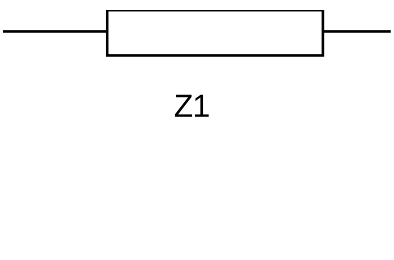
$$\begin{bmatrix} -j0,2 & 0,1 & -j0,9 \\ -j0,9 & -j0,2 & 0,1 \\ 0,1 & -j0,9 & -j0,2 \end{bmatrix}\begin{bmatrix} j0,2 & j0,9 & 0,1 \\ 0,1 & j0,2 & j0,9 \\ j0,9 & 0,1 & j0,2 \end{bmatrix} = \begin{bmatrix} 0,86 & 0,18-j0,07 & 0,18+j0,07 \\ 0,18+j0,07 & 0,86 & 0,18-j0,07 \\ 0,18-j0,07 & 0,18+j0,07 & 0,86 \end{bmatrix}$$

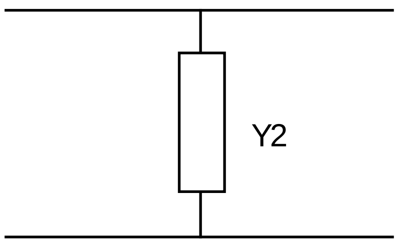
This component is linear, non matched, lossy, and non reciprocal. It has symmetry of order three



We see that most of the signal incoming in 1 flows to 3, with only a small proportion flowing to 2 or being reflected. From 2, most of the signal flows to 1, and from 3 to 2. This element circulates thus the signal between the ports, and it is indeed a not too good circulator (because not matched and having some signal flowing in the opposite direction to the main flow).

Problem 3
Prove that the scattering matrix of the following circuits





are given by:

$$[S]=\begin{bmatrix}\Gamma_1 & 1-\Gamma_1\\ 1-\Gamma_1 & \Gamma_1\end{bmatrix} \quad et \quad [S]=\begin{bmatrix}\Gamma_2 & 1+\Gamma_2\\ 1+\Gamma_2 & \Gamma_2\end{bmatrix}$$

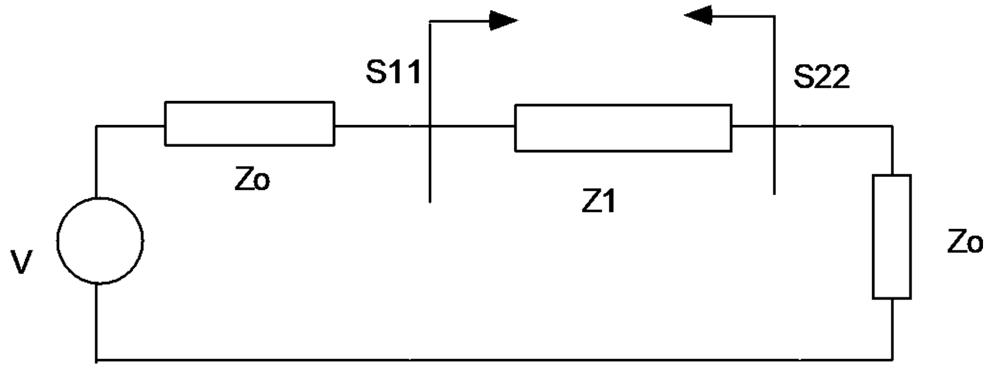
with

$$\Gamma_1=\frac{Z_1}{Z_1+2Z_0} \qquad et \qquad \Gamma_2=-\frac{Y_2}{Y_2+2Y_0}$$

The characteristic impedance at the reference planes is Zo

Solution:

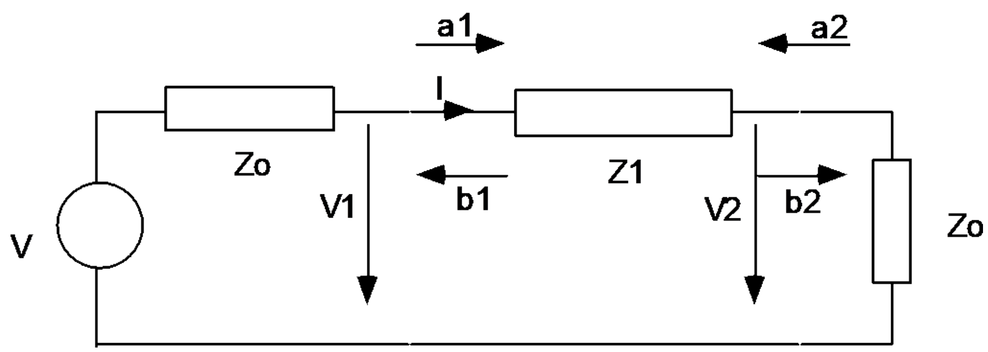
a) Terms S11 and S22 of the scattering matrix are obtained considering the following equivalent circuit:



We find immediately

$$s_{11}=s_{22}=\frac{(Z_1+Z_0)-Z_0}{(Z_1+Z_0)+Z_0}=\frac{Z_1}{Z_1+2Z_0}$$

Terms S12=S21 are obtained using the same circuit and applying the definition of the generalized wave amplitudes:



We write:

$$s_{21} = \frac{b_2}{a_1} = \frac{U_2 - Z_0 I_2}{U_1 + Z_0 I_1} = \frac{V_2 + Z_0 I}{V_1 + Z_0 I} = \frac{V - \dfrac{V\left(Z_0 + Z_1\right)}{Z_1 + 2Z_0} + Z_0 \dfrac{V}{Z_1 + 2Z_0}}{V - \dfrac{VZ_0}{Z_1 + 2Z_0} + Z_0 \dfrac{V}{Z_1 + 2Z_0}}$$

$$= \frac{Z_1 + 2Z_0 - Z_0 - Z_1 + Z_0}{Z_1 + 2Z_0 - Z_0 + Z_0} = \frac{2Z_0}{Z_1 + 2Z_0}$$

b) The solution is the dual of case a), and is obtained in the same way.